

CHAPTER IV

4.1 Introduction

In this chapter we see about barbell graph, friendship graph and path related graph

4.2 Main Results :

4.2.1 BARBELL GRAPH

A barbell (B_n) graph admits an even vertex odd edge root square mean labeling.

Proof:

Let G be a graph B_n where $V(G) = \{u_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i, u_{i+1} : 1 \leq i \leq n-1\}$.

We shall prove that B_n admits an even vertex odd edge root square mean labeling.

A barbell graph B_n is a graph with n vertices $\{u_1, u_2, \dots, u_n\}$ and n edges $\{u_1u_2, u_2u_3, \dots, u_nu_{n+1}\}$.

Assign each vertex is an even number $f(v_i) = 2i$, for $i = 0, 1, 2, \dots, n-1$. This gives all vertex labels are even.

Each edge $e = (v_i, v_{i+1})$ is labeled as $f(e_i) = \sqrt{f(v_i)^2 + f(v_{i+1})^2} / 2$ where $f(v_i)$ and $f(v_{i+1})$ are the labels of the connected edges.

Since adjacent vertex labels are consecutive even numbers, their squared sum always results in an odd value, ensuring that $f(e_i)$ remains odd.

Hence B_n graph satisfies that the even vertex odd edge root square mean labeling.

FRIENDSHIP GRAPH

A friendship (F_{2n}) graph admits an odd vertex even edge root square mean labeling.

Proof

Let G be a graph F_{2n} where $V(G) = \{u_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i, u_{i+1} : 1 \leq i \leq n-1\}$.

We shall prove that F_{2n} admits an odd vertex even edge root square mean labeling.

A friendship F_{2n} is a graph with n vertices $\{u_1, u_2, \dots, u_n\}$ and n edges $\{u_1u_2, u_2u_3, \dots, u_nu_{n+1}\}$.

Assign each vertex is an odd number $f(v_i) = 2i-1$, for $1 \leq i \leq n$. This gives all vertex labels are odd.

Each edge $e = (v_i, v_{i+1})$ is labeled as $f(e_i) = \sqrt{f(v_i)^2 + f(v_{i+1})^2} / 2$ where $f(v_i)$ and $f(v_{i+1})$ are the labels of the connected edges.

Since adjacent vertex labels are consecutive odd numbers, their sum two odd squares is always even.

Hence F_{2n} graph satisfies that the odd vertex even edge root square mean labeling.

PATH RELATED GRAPH

$P_n \odot K_{1,3}$ is even vertex odd edge root square mean labeling graph.

Proof:

Let $G = P_n \odot K_{1,3}$

Let $V(G) = \{u_i, u'_i, u''_i, u'''_i; 1 \leq i \leq n\}$ and $E(G) = \{u_i u'_i, u_i u''_i, u_i u'''_i; 1 \leq i \leq n \text{ \& } u_i u'_{i+1}; 1 \leq i \leq n-1\}$

Define a map $f: V(G) \rightarrow \{0, 1, 2, \dots, 2q\}$ by

$$f(u_i) = 8i-6; 1 \leq i \leq n$$

$$f(u'_i) = 8(i-1); 1 \leq i \leq n$$

$$f(u''_i) = 8i-4; 1 \leq i \leq n$$

$$f(u'''_i) = 8i-2; 1 \leq i \leq n$$

We denote the edge labels are both odd and distinct.

Hence $P_n \odot K_{1,3}$ is an even vertex odd edge root square mean labeling.