Corollary ①

If G is a simple planar graph then G has a vertex v of degree less than 6.

ie; there is a vertex v in G with $d(v) \leq 5$

Proof :

→ If G has only one vertex, this vertex must have degree zero.

→ If G has only two vertices, then both have degree atmost one.

→ Suppose that G has n vertices and $n \geq 3$

ie; G has atleast 3 vertices

Assume the degree of every vertex of G is atleast 6 [ie: $d(v) \geq 6$, for all vertex v in G]

Then $\sum_{v \in V[G]} d(v) = d(v_1) + d(v_2) + \cdots + d(v_n) \geq 6 + 6 + \cdots + 6$ (n times)

$$\sum_{v \in V[G]} d(v) \geq 6n \text{ ——— } (1)$$

If G has e edges then we have

$$\sum_{v \in V[G]} d(v) = 2e. \quad \text{———} \ (2).$$

Put (2) in (1) $\Rightarrow$ $2e \geq 6n.$

$$\underline{\underline{e \geq 3n.}} \longleftarrow (3)$$

We have, by above theorem $e \leq 3n-6.$

ie; $\underline{\underline{e < 3n}}$ —— (4)

Using (3) and (4) we get a contradiction.

$\therefore$ Our assumption is wrong

Thus there is atleast one vertex v of G such that $d(v) \not\geq 6$

$\therefore d(v) \leq 5$

Hence G has a vertex v of degree less than 6.

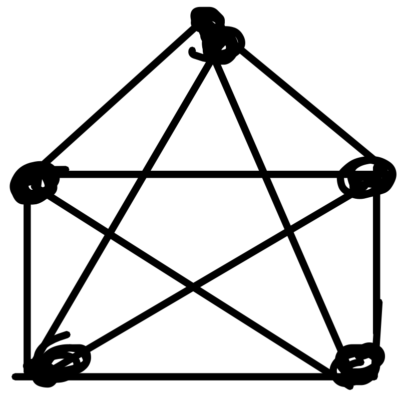
Corollary-2:

K_5 is non-planar

Proof:

We have K_5 is a Complete graph with 5 vertices. $(n=5)$

The number of edges e of a Complete graph with n vertices K_n is $e = \frac{n(n-1)}{2}$



K_5

$\therefore$ Total number of edges e of K_5 is

$$e = \frac{5(5-1)}{2} = 10$$

Now

$$3n-6 = 3(5)-6 = 9$$

ie; $9 < 10$

$3n-6 < e$.

$e \nleq 3n-6$

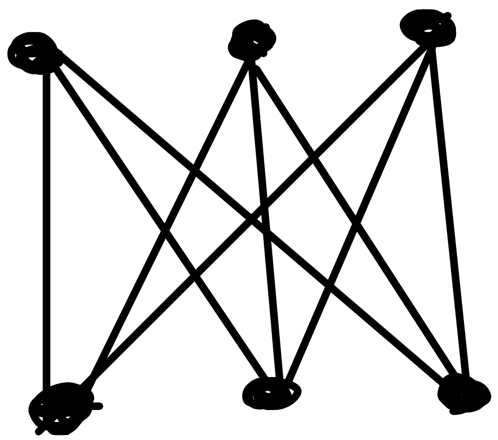
we have a simple graph G with n vertices and e edges is planar then $e \leq 3n-6$

But here $e \nleq 3n-6$

$\therefore$ G is not planar

ie; K_5 is not planar

Corollary-3 : $K_{3,3}$ is nonplanar.



Proof :

we have the bipartite graph $K_{3,3}$ contains no odd cycle.

So in Particular $K_{3,3}$ has no cycles of length 3.

Every face of a plane drawing of $K_{3,3}$ if such exists, must have atleast 4 boundary edges.

Hene $n=6$, $e=9$ $\Rightarrow$ Assume $K_{3,3}$ is planar

$n-e+f=2 \Rightarrow 6-9+f=2$

$$f=5 \quad \text{--- } ①$$

We have $4f \leq 2e \Rightarrow 2f \leq e$. [by proof of above then]

$$2f \leq 9$$

$$f \leq \frac{9}{2} \quad \text{--- } ②$$

Using ① and ② $\Rightarrow$ a contradiction

$\therefore K_{3,3}$ is not planar