A NOTE ON GTMS SPACE

Project Report submitted to Ayya Nadar Janaki Ammal College, Sivakasi.

in partial fulfillment of the requirements for the Degree of

Master of Science

in

MATHEMATICS

By

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(Reg. No. : 19PM37)

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Certificate

This is to certify that this project report entitled **"A NOTE ON GTMS SPACE"** being submitted by **Mr. M . SANGILIKUMAR (Reg. No.: 19PM37)**, final year student of M.Sc. degree course in Mathematics, Ayya Nadar Janaki Ammal College (Autonomous), Sivakasi, affiliated to Madurai Kamaraj University, Madurai, is a bonafide record of work carried out by him under the guidance and supervision of **Dr. V. SUTHA DEVI**, Assistant Professor, Department of Mathematics (U.G.), Ayya Nadar Janaki Ammal College (Autonomous), Sivakasi.

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It is further certified that to the best of my knowledge, this project report or any part thereof has not been submitted in this College or elsewhere for the award of any other degree or diploma.

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Declaration

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I further declare that this report or any part thereof has not been submitted in this College or elsewhere for any other degree or diploma.

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"Nantri marapathu nanranru"

-Thiruvalluvar.

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(M.SANGILIKUMAR)

Contents

Chapter 1

INTRODUCTION

T.Nohri and V.Popa established the notion of minimal structure in 2000.They also established the notion of m_Y-open sets and m_Y-closed sets. They characterzied those sets using m_Y-closure and $m_Y - interior$ operators. T.Nohri and V.Popa introduce the definitions and characterizations of seperation axioms by using the concept of minimal structure. C.Boonpok established the concept of binomial structure spaces and studied m_Y^1 m_Y^2-closed sets and m_Y^1 $m_Y^2 - opensets$ in binomial stucture spaces.

A.Csaszar introduced the notion of generalised neighborhood systems and generalised topological spaces.He established the notions of continuous functions and associated interior and closure operators on generalised neighborhood systems and generalised topological spaces .Further he studied seperation axioms for generalised topologies in C.Boonpok established the concept of bigeneralised topological spaces and (m, n)-closed sets and (m, n)-open sets in bigeneralised topologicalspaces.In this project we study a new space which consists of a set X, *Generalised topology* and *minimal structure spaces* on Y (GTMS space). We introduced the concepts of seperation aioms on generalised topology and minimal structure spaces.

In chapter 2 , we recall the definitions and defines GTMS space.

In chapter 3 , we define the $m_Y - opensets$ and $m_Y - closedsets$ with their properties.

In chapter 4 , we define the separation axioms with their properties.

Finally we conclude that the properties of T_2 Hausdorff GTMS space and $T_{1/2}$-GTMS space.

Chapter 2

PRELIMINARIES

In this section we recall some preliminary definitions which are required in the generalized topological space.

Definition 2.1. *A topology on a set Y is a collection $\Im$ of subsets of Y having the following properties:*
(1) $\emptyset$ and Y are in $\Im$.
(2) The union of the elements of any subcollection of $\Im$ is in $\Im$.
(3) The intersection of the elements of any finite subcollection of $\Im$ is in $\Im$.

Example 2.1. *Let $Y = \{1,2,3\}$ and $\Im = \{\emptyset, X, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}\}$. Let $J = \{1,3\}$, $K = \{2,3\}$. Here $J \in \Im$ and $K \in \Im$ but $J \cap K \notin \Im$.*
Hence $\Im$ is not a topology on Y.

Definition 2.2. *Let X and Y be topological space.The product topology on X $\times$ Y is the topology having as basis the collection **B** of all sets of the form O $\times$ P , where O is an open subset of X and P is an open subset of Y.*

Definition 2.3. *A subset J of a topological space Y is said to be closed if the set Y-J is open.*

Definition 2.4. *Let J be a nonempty subset of a topological space $(Y, \Im)$ and $x \in J$.Then x is said to be an interior point of J if there exist an open set O such that $x \in O$ and $O \subseteq J$.*
The collection of all interior point of J denoted by Int(J).

Definition 2.5. *Let J be a nonempty subset of a topological space* $(Y, \Im)$ *and* $x \in J$*.Then x is said to be an accumulation point of J if for each open set O containing x,* $O \cap (J\text{-}\{x\})$ *is nonempty.*

Definition 2.6. *For* $J \subseteq Y$*, the derived set of J denoted by* $J^{'}$ *is defined as* $J^{'} = \{y \in Y : y$ *is a limit point of* $J\}$

Definition 2.7. *For* $J \subseteq Y$*, the closure of J denoted by Cl(J), is defined as* $Cl(J) = J \cup J^{'}$.

Note 1. *For any subset J of a topological space (Y,* $\Im$*), Int(J) is the largest open set contained in J.*

Note 2. *For any subset J of a topological space (Y,* $\Im$*), Cl(J) is the smallest closed set containing in J.*

Note 3. *For any subset J of a topological space Y, Cl(J) is a closed set and J is a subset of Cl(J).*

Definition 2.8. *A sequence* $\{y_n\}$ *in a topological space (Y,* $\Im$*) is said to converge to a point* $y \in Y$ *if for each open set O containing y there exists* $n_0 \in N$ *such that* $y_n \in O$, $\forall\ n \geq n_0$.

Chapter 3

GENERALIZED TOPOLOGY AND MINIMAL STRUCTURE SPACE

In this section, we introduced the notions of generalized topology and minimal structure spaces.

Definition 3.1. *Let $Y \neq \emptyset$ and let $\Im$ be a collection of subsets of Y .Then $\Im$ is called generalised topology (briefly GT) on Y if and only if it satisfies the following conditions :*
i) $\emptyset \in \Im$.
ii) $G_i \in \Im$ *for* $i \in I$ *implies* $\bigcup_i G_i \in \Im$.
We call this pair $(Y, \Im)$ a generalised topological space on X.
The elements of $\Im$ are called $\Im$-open sets.
The complements are called $\Im$-closed sets.
The closure of a subset J in generalised topological space $(Y, \Im)$,denoted by $\Im$ - Cl(J) , is the intersection of generalised closed sets including J.
Interior of J,denoted by $\Im$ - Int(J) , is the union of generalised open sets containing J.

Theorem 3.1. *Let (Y,$\Im$) be a generalised topological space . Then*
i) $\Im$ - Cl(J) = X - $\Im$-Int(J);
ii) $\Im$-Int(J) = Y - $\Im$-Cl(J)

Proposition 3.1. *Let* $(Y, \Im)$ *be a generalized topological space and* $J \subseteq Y$*. Then*
i) $l \in g - Int(J)$ *if and only if* $\exists$ $M \in g$ *such that* $l \in M \subseteq J$
ii) $l \in g - Cl(J)$ *if and only if* $M \cap J \neq \emptyset$ *for every g-open set M containing l.*

Proposition 3.2. *Let* $(Y, \Im)$ *be a generalized topological space . For the subsets J and K we have*

1) $\Im$ *-Cl(Y - J) = Y -* $\Im$*-Int (J) and*
$\Im$ *- Int(Y-J) = Y - g-Cl(J);*
2) if Y-J $\in \Im$ *, then* $\Im$ *- Cl(J) = J and*
if J $\in g$*, then* $\Im$ *- Int(J) = J;*
3) if J $\subseteq$ *K , then* $\Im$ *- Cl(J)* $\subseteq$ $\Im$ *- Cl(K) and*
$\Im$ *- Int(J)* $\subseteq$ $\Im$ *- Int(K) ;*
4) J $\subseteq$ $\Im$ *- Cl(J) and*
$\Im$ *- Int(K)* $\subseteq$ *J*
5) $\Im$ *- Cl(* $\Im$ *- Cl(J)) =* $\Im$ *- Cl(J) and*
$\Im$ *- Int(* $\Im$ *-Int(J)) =* $\Im$ *- Int(J).*

Definition 3.2. *Let* $Y \neq \emptyset$ *and let* $\zeta(Y)$ *be the power set of* Y*. A subfamily* m_Y *of* $\zeta(Y)$ *is called a minimal structure (briefly m-structure) on Y if*
$\emptyset \in m_Y$ *and*
$Y \in m_Y$*.*
From (Y, m_Y) *, we denote a set* Y *which is nonempty and* m_Y *an m-structure on* Y *and is called an m-space .*
Each member of m_Y *is said to be* m_Y*-open and the complement of* m_Y*-open set is said to be* mY*-closed.*

Definition 3.3. *Let* $Y \neq \emptyset$ *and* m_Y *an m-structure on* Y*. We have for J is a subset of* Y *then the* m_Y*-closure of J is simply called as* m_Y*-cl(J) and* m_Y*-interior of J is simply called as* m_Y*-int(J) are defined by*

(I) $m_Y - cl(J) = \cap\{I : J \subseteq I, Y - I \in m_Y\}$

(II) m_Y*-Int (J)* $= \cap\{O{:}O \subseteq J$ *and* $O \in m_Y\}$.

Lemma 3.1. *Let* $Y \neq \emptyset$ *and take* m*-structure on* Y*.The following conditions hold for* J *is a subset of* Y *and for* K *is a subset of* Y*,*

(1) m_Y *- Cl(Y-J) = Y-*m_Y*-Int(J) and*

m_Y*-Int(Y-J)=Y-*m_Y*-Cl(J),*

(2) if Y*-*$J \in m_Y$*, then* m_Y*-Cl(J)=J and*

if $J \in m_Y$ *then* m_Y*-Int(J)=J,*

(3) if $J \subseteq K$ *, then* m_Y*-Cl(J)* $\subseteq$ m_Y*-Cl(K) and*

m_Y *- Int(J)* $\subseteq$ m_Y *- Int(K),*

(4) m_Y*-Cl(*$\emptyset$*)=*$\emptyset$*,*

m_Y*-Cl(Y)=Y,*

m_Y*-Int(*$\emptyset$*)=*$\emptyset$ *and*

m_Y*-Int(Y)=Y,*

(5) $J \subseteq$ m_Y*-Cl(J) and*

m_Y*-Int(J)*$\subseteq$ *J ,*

(6) m_Y*-Cl (*m_Y*-Cl(J)) =* m_Y*-Cl(J) and*

m_Y*-Int (*m_Y*-Int(J)) =* m_Y*-Int(J).*

Lemma 3.2. g_Y*-Cl (* $\emptyset$ *) =* $\emptyset$*.*

Proof

Let (Y,g_Y*,*m_Y*) be a generalized topology and minimal structure space.*

We have Empty set is closed in a topological space.

Empty set is closed in T.

It follows that the closed set equals to its closure.

Lemma 3.3. *g_Y-Int($\emptyset$) $= \emptyset$.*

Proof

Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.

Then empty set is a open set .

Since empty set has no points.

Then it vacuously satisfies interior definition. Thus g_Y-Int($\emptyset$) $= \emptyset$.

Lemma 3.4. *g_Y-Int (Y) = Y.*

Proof

Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.

Let Y be an open set.

For each $y \in Y$, Y is an open neighbourhood of y.

This implies that For each point $y \in Y$, is an interior point of Y.

Thus g_Y-Int(Y) = Y.

Lemma 3.5. *g_Y-Cl ($\emptyset$) $= \emptyset$.*

Proof

Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.

We have Empty set is closed in a topological space.

Empty set is closed in T.

It follows that the closed set equals to its closure.

Lemma 3.6. *g_Y-Int($\emptyset$) $= \emptyset$.*

Proof

Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.

Then empty set is a open set .

Since empty set has no points.

Then it vacuously satisfies interior definition.

Thus g_Y-Int($\emptyset$) $= \emptyset$.

Lemma 3.7. *g_Y-Int (Y) = Y.*

Proof

Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.

Let Y be an open set.

For each $y \in$ Y, Y is an open neighbourhood of y.

This implies that For each point $y \in$ Y, is an interior point of Y.

Thus g_Y-Int(Y) = Y.

Definition 3.4. *Let Y be a nonempty set and let g_Y be a generalized topology and m_Y a minimal structure on Y.*

A triple (Y,g_Y,m_Y) is called a generalized topology and minimal structure space.

Let (Y,g_Y,m_Y) is called a generalized topology and minimal structure space and J is a subset of Y

The closure of J in g_Y is denoted by g_Y-Cl(J).

The interior of J in g_Y is denoted by g_Y-Int(J).

The closure of J in m_Y is denoted by m_Y-Cl(J).

The interior of J in m_Y is denoted by m_Y-Int(J).

Definition 3.5. *Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space.*

A subset J of Y is said to be $g_Y m_Y$-closed set if g_Y-Cl (m_Y-Cl(J)) = J.

Definition 3.6. *A subset J of Y is said to be $m_Y g_Y$-closed set if*

$m_Y - Cl(g_Y - Cl(J)) = J$

The complement of a $g_Y m_Y$ - closed set is called $g_Y m_Y$ - open set.

The complement of a $m_Y g_Y$ - closed set is called $m_Y g_Y$ - open set.

Lemma 3.8. *Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space and let J is a subset of Y . Then J is $g_Y m_Y$ - closed set if and only if*

m_Y-*Cl(J)* = *J and* g_Y-*Cl(J)* = *J.*

Proof

Let (Y,g_Y*,*m_Y*) be a generalized topology and minimal structure space.*

Let us assume that J is $g_Y m_Y$ *- closed set.*

Then g_Y-*Cl(* m_Y-*Cl(J))* = *J.*

Since $m_Y \subseteq g_Y$-*Cl(* m_Y-*Cl(J))* = *J, we have* m_Y-*Cl(J)* = *J .*

From J $\subseteq m_Y$-*Cl(J),*

we have g_Y-*Cl(J)* $\subseteq g_Y$-*Cl(* m_Y-*Cl(J))* = *J.*

Hence g_Y-*Cl(J)* = *J.*

Conversely,

Let J is a subset of Y such that m_Y-*Cl(J)* = *J and* g_Y-*Cl(J)* = *J.*

Then J = g_Y-*Cl(J)* = g_Y-*Cl(*m_Y-*Cl(J)).*

Hence J is $g_Y m_Y$ *- closed set.*

Lemma 3.9. *Let (Y,*g_Y*,*m_Y*) be a generalized topology and minimal structure space and let J is a subset of Y.Then J is* $m_Y g_Y$ *- closed set if and only if* m_Y-*Cl(J)* = *J and* g_Y-*Cl(J)*= *J.*

Proof

Let (Y,g_Y*,*m_Y*) be a generalized topology and minimal structure space.*

Let us assume that J is $m_Y g_Y$*-closed set.*

Then m_Y-*Cl(* g_Y-*Cl(J))* = *J.*

Since $m_Y \subseteq m_Y$-*Cl(* g_Y-*Cl(J))* = *J, we have* m_Y-*Cl(J)* = *J .*

From J $\subseteq m_Y$-*Cl(J),*

we have g_Y-*Cl(J)* $\subseteq m_Y$-*Cl(* g_Y-*Cl(J))* = *J.*

Hence g_Y-*Cl(J)* = *J.*

Conversely,

Let J is a subset of Y such that m_Y-*Cl(J)* = *J and* g_Y-*Cl(J)* = *J .*

Then $J = g_Y\text{-}Cl(J) = m_Y\text{-}Cl(\ g_Y\text{-}Cl(J)\ )$.

Hence J is $m_Y g_Y$ *- closed set.*

Proposition 3.3. *Let* (Y,g_Y,m_Y) *be a generalized topology and minimal structure space and let J is a subset of Y . Then J is* $g_Y m_Y$*-closed set if and only if* $m_Y g_Y$*-closed set .*

Proof

Let us assume that J is $g_Y m_Y$ *- closed set.*

To prove,$m_Y g_Y$ *- closed set.*

By ***lemma 3.8****,*

"Let (Y,g_Y,m_Y) *is called a generalized topology and minimal structure space and let J is a subset of Y.Then J is* $g_Y m_Y$ *- closed set if and only if*

$m_Y\text{-}Cl(J) = J$ *and* $g_Y\text{-}Cl(J)= J$*"*

Then We have,

$m_Y\text{-}Cl(J) = J$ *and* $g_Y\text{-}Cl(J)= J$.

Hence by ***lemma 3.9****,*

"Let (Y,g_Y,m_Y) *is called a generalized topology and minimal structure space and let J is a subset of Y.Then J is* $m_Y g_Y$ *- closed set if and only if*

$m_Y\text{-}Cl(J) = J$ *and* $g_Y\text{-}Cl(J)= J$*"*

From these lemma we can prove that ,

J is $m_Y g_Y$ *- closed set.*

Definition 3.7. *Let* (Y,g_Y,m_Y) *be a generalized topology and minimal structure space and let J is a subset of Y.*

Then is said to be Closed if J is $g_Y m_Y$ *- closed set.*

The complement of a closed set is called an $g_Y m_Y$ *- open set.*

Definition 3.8. *Let* (Y,g_Y,m_Y) *be a generalized topology and minimal structure space and let J is a subset of Y.*

Then J is Closed if and only if J is $m_Y g_Y$ - closed set.

Proposition 3.4. *Let (Y, g_Y, m_Y) be a generalized topology and minimal structure space.If J and K are closed, then $J \cap K$ is closed.*

Proof

Let us assume that J and K are closed.

Then g_Y-Cl(m_Y-Cl(J)) = J and g_Y-Cl(m_Y-Cl(K)) = K.

Since $J \cap K \subseteq J$ and $J \cap K \subseteq K$, also we have

m_Y-Cl($J \cap K$) $\subseteq$ m_Y-Cl(J) and m_Y-Cl($J \cap K$) $\subseteq$ m_Y-Cl(K).

Thus g_Y-Cl(m_Y-Cl($J \cap K$)) $\subseteq$ g_Y(m_Y-Cl(J)) = J

Also g_Y-Cl(m_Y-Cl($J \cap K$)) $\subseteq$ g_Y(m_Y-Cl(K)) = K.

Hence g_Y-Cl(m_Y-Cl($J \cap K$)) $\subseteq J \cap K$.

Since $J \cap K \subseteq$ m_Y-Cl($J \cap K$) $\subseteq$ g_Y-Cl(m_Y-Cl($J \cap K$)),

We have g_Y-Cl(m_Y-Cl($J \cap K$)) = $J \cap K$.

Therefore, $J \cap K$ is $g_Y m_Y$-closed.

Hence $J \cap K$ is closed.

Note 4. *In general,the union of two closed sets is not a closed set. This can be seen from the following example.*

Example 3.1. *Let Y={1,2,3,4}.*

We define generalized topology g_Y and minimal structure m_Y on Y as follows:

g_Y= { $\emptyset$, {3,4} , {1,3,4} , {2,3,4} } and

m_Y = { $\emptyset$, {3,4} , {1,3,4} , {2,3,4} , Y }.

Then {1} and {2} are closed but the union of {1} and {2} is equal to {1,2}, which is not closed.

Proposition 3.5. *Let (Y, g_Y, m_Y) be a generalized topology and minimal structure space.*

Then J is open if and only if J = g_Y*-Int(* m_Y*-Int(J)).*

Proof

Let us assume that J is open.

Then J^c *is closed and also it satisfies* g_Y*-Cl(* m_Y*-Cl(*J^c*)) =* J^c*.*

g_Y*-Cl(Y −* m_Y*-Int(J))=* J^c*.*

Thus Y − g_Y*-Int(* m_Y*-Int(J)) = Y−J.*

Hence g_Y*-Int(* m_Y*-Int(J)) = J.*

Coversely assume that ,

Let J is a subset of Y such that J = g_Y*-Int(* m_Y*-Int(J)).*

Then J^c *= Y −* g_Y*-Int(* m_Y*-Int(J)).*

By proposition 2.2,

J^c *=* g_Y*-Int(* m_Y*-Int(J)).*

Thus J^c *=* g_Y*-Cl(* m_Y*-Cl(*J^c*)).*

Hence J^c *is closed.*

This implies that J is open.

Proposition 3.6. *Let (Y,*g_Y*,*m_Y*) be a generalized topology and minimal structure space.*

If J and K are open,then $J \cup K$ *is open.*

Proof*.*

Let us assume that J and K are open.

Then g_Y*-Int(* m_Y*-Int(J)) = J and* g_Y*-Int(* m_Y*-Int(K)) = K.*

J is a subset of $J \cup K$ *and K is a subset of* $J \cup K$*,*

We have , m_Y*-Int(J) is a subset of* m_Y*-Int(* $J \cup K$ *) and*

m_Y*-Int(K) is a subset of* m_Y*-Int(* $J \cup K$ *).*

Thus J= g_Y*-Int(* m_Y*-Int(J))* $\subseteq$ g_Y*-Int(* m_Y*-Int(*$J \cup K$ *)) and*

K = g_Y*-Int(* m_Y*-Int(K))* $\subseteq$ g_Y*-Int(* m_Y*-Int(*$J \cup K$*)).*

Hence $J \cup K \subseteq g_Y$*-Int(* m_Y*-Int(*$J \cup K$*)).*

But g_Y*-Int(* m_Y*-Int(*$J \cup K$*))* $\subseteq m_Y$*-Int(* $J \cup K$ *)* $\subseteq J \cup K$.

From this we obtain that $J \cup K = g_Y$*-Int(* m_Y*-Int(*$J \cup K$*)).*

Hence $J \cup K$ *is open.*

Note 5. *The intersection of two open sets is not a open set.*

It can be seen from the following example.

Example 3.2. *Let* $Y=\{1,2,3\}$.

We define generalized topology g_Y *and minimal structure* m_Y *on* Y *as follows:*

$g_Y = \{ \emptyset , \{2\} ,\{3\} , \{1,2\} ,\{1,3\} , \{2,3\} , Y \}$ *and*

$m_Y = \{ \emptyset , \{1\} , \{3\} , \{1,2\} , \{1,3\} , Y \}$.

Then $\{1,2\}$ *and* $\{1,3\}$ *are open .*

But the intersection of $\{1,2\}$ *and* $\{1,3\}$ *is* $\{1\}$ *, which is not open.*

Chapter 4

CLOSURE AND INTERIOR ON GTMS SPACE

In this section, we introduce the concepts of closure and interior of GTMS space with s-open, s-closed sets and c-open, c-closed sets.

Definition 4.1. *Let (Y, g_Y, m_Y) be a GTMS space and let J is a subset of Y. Then J is said to be s-closed if g_Y-$Cl(J)$ = m_Y-$Cl(J)$.*

Definition 4.2. *Let (Y, g_Y, m_Y) be a GTMS space and let J is a subset of Y. Then J is said to be c-closed if g_Y-$Cl(\ m_Y\text{-}Cl(J)\ )$ = m_Y-$Cl(\ g_Y\text{-}Cl(J)\ )$.*

Definition 4.3. *The complement of s-closed is said to be a s-open set. The complement of c-closed is said to be a c-open set.*

Proposition 4.1. *Let (Y, g_Y, m_Y) be a GTMS space and let J is a subset of Y.*
If J is closed , then J is s-closed .

Proof

Let us assume that J is closed.

Then J is $g_Y m_Y$-closed.

By lemma ,

Let (Y, g_Y, m_Y) be a generalized topology and minimal structure space and let J is a subset of Y.Then J is $g_Y m_Y$-closed set if and only if m_Y-$Cl(J)$ = J and g_Y-$Cl(J)$= J.

We have , m_Y-$Cl(J)$ = J and g_Y-$Cl(J)$= J.

Thus g_Y*-Cl(J)* = m_Y*-Cl(J).*

Hence J is J is s-Closed.

Note 6. *The converse of the above proposition is not true.*

It can be seen from the following example.

Example 4.1. *Let* $Y = \{1,2,3,4\}$.

We define generalized topology g_Y *and minimal structure* m_Y *on Y as follows:*

$g_Y = \{ \emptyset , \{1\} , \{1,3\} \}$ *and*

$m_Y = \{ \emptyset , \{1\} , \{2,3\} , Y \}$.

Then g_Y-$Cl(\{3\}) = \{2,3\} = m_Y$-$Cl(\{3\})$.

But g_Y-$Cl(m_Y$-$Cl(\{3\}) = \{2,3\} \neq \{3\}$.

Proposition 4.2. *Let* (Y,g_Y,m_Y) *be a generalized topology and minimal structure space and let J is a subset of Y.*

If J is s-closed, then J is c-closed .

Proof

Let us assume that J is s-closed.

Then g_Y-$Cl(J) = m_Y$-$Cl(J)$.

This implies that

g_Y-$Cl(m_Y$-$Cl(J)) = g_Y$-$Cl(g_Y$-$Cl(J)) = g_Y$-$Cl(J) = m_Y$-$Cl(J) = m_Y$-$Cl(m_Y$-$Cl(J)) = m_Y$-$Cl(g_Y$-$Cl(J))$.

Hence J is c-closed .

Example 4.2. *Let* $Y = \{1,2,3\}$.

We define generalized topology g_Y *and minimal structure* m_Y *on Y as follows:*

$g_Y = \{ \emptyset , \{1\} , \{1,3\} \}$ *and*

$m_Y = \{ \emptyset , \{2\} , \{2,3\} ,Y \}$.

Then g_Y-$Cl(\{3\}) = Y = m_Y$-$Cl(\{3\})$.

But g_Y-$Cl(\{3\}) = \{2,3\} \neq \{3\} = m_Y$-$Cl(\{3\})$.

Proposition 4.3. *Let (Y,g_Y,m_Y) be a generalized topology and minimal structure space and let J is a subset of Y.Then*
(1) J is s-open if and only if
g_Y-*Int*(J) = m_Y-*Int*(J),
(2) J is c-open if and only if
g_Y-*Int*(m_Y-*Int*(J)) = m_Y-*Int*(g_Y-*Int*(J)).

Proposition 4.4. *Let (Y,g_Y,m_Y) be a GTMS space and let J is a subset of Y. If J is open, then J is s-open .*

Proposition 4.5. *Let (Y,g_Y,m_Y) be a GTMS space and let J is a subset of Y. If J is s-open , then J is c-open .*

Lemma 4.1. *Let (Y,g_Y,m_Y) be a GTMS space and let J is a subset of Y. If J is s-closed , then g_Y-Cl(J) is closed .*

Proof

Let us assume that J is s-Closed.
Then g_Y-Cl(J) = m_Y-Cl(J).
This implies that
m_Y-*Cl*(g_Y-*Cl*(J)) = m_Y-*Cl*(m_Y-*Cl*(J)) = m_Y-*Cl*(J) = g_Y-*Cl*(J).
Thus g_Y-*Cl*(m_Y-*Cl* (g_Y-*Cl*(J))) = g_Y-*Cl*(g_Y-*Cl*(J)).
g_Y-*Cl*(g_Y-*Cl*(J)) = g_Y-*Cl*(J).
Hence, g_Y-*Cl*(J) *is closed .*

Lemma 4.2. *Let (Y, g_Y, m_Y) be a GTMS space and let J is a subset of Y. If J is s-closed , then m_Y-Cl(J) is closed .*

Proof

Let us assume that J is s-Closed.

Then g_Y-Cl(J) = m_Y-Cl(J).

This implies that

g_Y-Cl(m_Y-Cl(J)) = g_Y-Cl(g_Y-Cl(J)) = g_Y-Cl(J) = m_Y-Cl(J).

Thus m_Y-Cl(g_Y-Cl(m_Y-Cl(J))) = m_Y-Cl(m_Y-Cl(J)).

m_Y-Cl(m_Y-Cl(J)) = m_Y-Cl(J).

Hence, m_Y-Cl(J) *is closed .*

Theorem 4.1. *Let (Y, g_Y, m_Y) be GTMS space and let J is a subset of Y. Then J is closed if and only if there exists a s-closed set K such that g_Y-Cl(K) = J.*

Proof

Let us assume that J is closed.

Thus J is s-Closed and

J = g_Y-Cl(J)=m_Y-Cl(J).

Take K = J.

Then K is s-closed and J = g_Y-Cl(K).

Coversely,

Let us assume that there exists a s-closed set K such that

g_Y-Cl(K) = J.

By lemma 4.1,

"Let (Y, g_Y, m_Y) be a generalized topology and minimal structure space and let J is a subset of Y".

If J is s-closed , then g_Y-Cl(J) is closed ".

Hence $J = g_Y$*-Cl(K) is closed.*

Lemma 4.3. *Let* (Y, g_Y, m_Y) *be a GTMS space and let J is a subset of Y. If J is c-closed, then* g_Y*-Cl(* m_Y*-Cl(J)) is closed.*

Proof

Let us assume that J is c-closed.

Take $K = g_Y$*-Cl(* m_Y*-Cl(J)).*

Since m_Y*-Cl(K) =* m_Y*-Cl(* m_Y*-Cl(*g_Y*-Cl(J))) =* m_Y*-Cl(* g_Y*-Cl(J)),*

Then we have g_Y*-Cl(* m_Y*-Cl(K)) =* g_Y*-Cl(* m_Y*-Cl(* g_Y*-Cl(J))) =* g_Y*-Cl(* g_Y*-Cl(* m_Y*-Cl(J))) =* g_Y*-Cl(* m_Y*-Cl(J)) = K.*

Hence g_Y*-Cl(*m_Y*-Cl(J)) = K is closed.*

Lemma 4.4. *Let* (Y, g_Y, m_Y) *be a GTMS space and let J is a subset of Y. If J is c-Closed, then* m_Y*-Cl(* g_Y*-Cl(J)) is closed.*

Proof

Let us assume that J is c-closed.

Take $K = m_Y$*-Cl(* g_Y*-Cl(J)).*

Since g_Y*-Cl(K) =* g_Y*-Cl(* g_Y*-Cl(* m_Y*-Cl(J))) =* g_Y*-Cl(* m_Y*-Cl(J)),*

Then we have

m_Y*-Cl(* g_Y*-Cl(K)) =* m_Y*-Cl(* g_Y*-Cl (* m_Y*-Cl(J))) =* m_Y*-Cl(* m_Y*-Cl(* g_Y*-Cl(J))) =* m_Y*-Cl(* g_Y*-Cl(J)) = K.*

Hence m_Y*-Cl(* g_Y*-Cl(J)) = K is closed.*

Theorem 4.2. *Let* (Y, g_Y, m_Y) *be a GTMS and let J is a subset of Y. Then J is closed if and only if there exists a c-closed set K such that J =* g_Y*-Cl(* m_Y*-Cl(K)) .*

Proof

Let us assume that J is closed.

Thus $J = g_Y$*-Cl(* m_Y*-Cl(K)) and J is c-closed.*

Take $K = J$.

Then K *is a c-closed set and* $J = g_Y\text{-}Cl(\ m_Y\text{-}Cl(K)\ )$.

Conversely,

Let us assume that there exists a c-closed set K *such that*

$J = g_Y\text{-}Cl(\ m_Y\text{-}Cl(K)\ )$.

By lemma 4.3,

"Let (Y, g_Y, m_Y) *be a generalized topology and minimal structure space and let* J *is a subset of* Y*".*

If J *is c-Closed, then* $g_Y\text{-}Cl(\ m_Y\text{-}Cl(J)\ )$ *is closed".*

From this we get $J = g_Y\text{-}Cl(\ m_Y\text{-}Cl(K)\ )$ *is closed.*

Chapter 5

SEPARATION AXIOMS IN GTMS SPACE

In this section we introduce the concepts of T_1-GTMS, T_2-GTMS and $T_{1/2}$-GTMS important properties, examples of a Hausdorff space and their characterizations.

Definition 5.1. *A GTMS space (Y,g_Y,m_Y) is called T_1-GTMS space if for any pair of distinct points x and y in Y,there exist a g_Y-open set O and a m_Y-open set P such that $x \in O$, $y \notin O$ and $y \in P$, $x \notin P$.*

Lemma 5.1. *Let (Y,g_Y,m_Y) be a GTMS space and let x,$y \in X$ be such that $x \neq y$.*
If (Y,g_Y,m_Y) is a T_1-GTMS space , then there exist a g_Y-open set O containing x but not y and a m_Y-open set P containing x but not y.

Theorem 5.1. *Let (Y,g_Y,m_Y) be a GTMS space. Y is a T_1-GTMS space if and only if every singleton subset of Y is closed.*

Proof

Let us assume that Y is a T_1-GTMS space.

Let $x \in Y$.

We want to show that $\{x\} = g_Y$-$Cl(\{x\})$ and $\{x\} = m_Y$-$Cl(\{x\})$.

Let $y \in Y$ be such that $y \neq x$.

By lemma 3.8,

"Let (Y,g_Y,m_Y) be a GTMS space and let x,$y \in X$ be such that $x \neq y$.

If (Y,g_Y,m_Y) is a T_1-GTMS space, then there exist a g_Y-open set O containing x but not y and a m_Y-open set P containing x but not y".

Then there exist a g_Y-open set O and a m_Y-open set P such that

$y \in O$, $x \notin O$ *and* $y \in P$, $x \notin P$.

Thus $O \cap \{x\} = \emptyset$ *and* $P \cap \{x\} = \emptyset$ *implies that*

$y \notin g_Y$-*Cl*($\{x\}$) *and* $y \notin m_Y$-*Cl*($\{x\}$).

Hence $\{x\} = g_Y$-*Cl*($\{x\}$) *and* $\{x\} = m_Y$-*Cl*($\{x\}$).

Therefore $\{x\}$ *is closed.*

Coversely,

Let us assume that every singleton subset of Y is closed.

Let x,$y \in Y$ *be such that* $x \neq y$.

By our assumption, we have

$\{x\} = m_Y$-*Cl*($\{x\}$) *and* $\{y\} = g_Y$-*Cl*($\{y\}$).

Since $x \notin g_Y$-*Cl*($\{y\}$) *and* $y \notin g_Y$-*Cl*($\{x\}$),

Then there exist a g_Y-open set O and m_Y-open set P such that $x \in O$, $O \cap \{y\} = \emptyset$ *and*

$y \in P$, $P \cap \{x\} = \emptyset$.

Then $x \in O$, $y \notin O$ *and* $y \in P$, $x \notin P$.

Hence Y is a T_1-GTMS space.

Definition 5.2. *A GTMS space (Y,g_Y,m_Y) is said to be a* ***Hausdorff*** *GTMS space or T_2-GTMS space if for any pair of distinct points x and y in Y.*

Then there exist a g_Y-open set O and m_Y-open set P such that $x \in O$, $y \in P$ *and* $O \cap P = \emptyset$.

Example 5.1. *If Y is a set containing atleast two elements $\emptyset$ and Y then the topological space is not Hausdorff.*

Example 5.2. *Let* $Y = \{1,2,3\}$.

Let $\Im = \{ Y , \emptyset , \{1\} , \{2\} , \{1,2\} \}$.

Then $(\Im , Y)$ *is not a Haudorff space.*

Note 7. *If* Y *is a* T_1*-GTMS space , and if* J *is a finite subset of* Y*, then* $J' = \emptyset$.

Theorem 5.2. *Every finite* T_1*-GTMS space is discrete.*

Proof

Let A *be a finite set.*

Let $T = (A, \Im)$ *be a* T_1*-GTMS space.*

Let O *be any subset of* A.

Let $H = c(O)$ *be the complement of* O *relative to* A *Then* $O = c(H)$.

H can be written as,

$H = \bigcup_x \{x\}$.

From the equivalent definition of T_1*-GTMS space,*

For all x *in* H *such that* $\{x\}$ *is closed in* T.

Since A *is finite, then* H *is a finite union of closed sets of* T.

Therefore, by definition of closed set,

$O = c(H)$ *is open in* T.

Since O *is arbitrary,*

It follows that for any subset O *of* A *such that* A *is in* $\Im$.

Hence every finite T_1*-GTMS space is discrete.*

Proposition 5.1. *Let* (Y, g_Y, m_Y) *be a GTMS space. If* Y *is a* T_2*-GTMS space , then* Y *is a* T_1*-GTMS space.*

Proof

Let us assume that Y *is a* T_2*-GTMS space.*

We want to prove that Y *is a* T_1*-GTMS space.*

By the definition of T_2-GTMS,

A GTMS space (Y,g_Y,m_Y) is said to be a ***Hausdorff*** *GTMS space or T_2-GTMS space if for any pair of distinct points x and y in Y.*

Then there exist a g_Y-open set O and m_Y-open set P such that $x \in O$, $y \in P$ and $O \cap P = \emptyset$".

As $O \cap P = \emptyset$, it follows from the definition of disjoint sets that

$x \in O$ implies $x \notin P$.

$y \in P$ implies $y \notin O$.

So if $x \in O$ and $y \in P$ then:

There exist $O \in Y$ such that $x \in O$ and $y \notin P$.

There exist $P \in Y$ such that $y \in P$ and $x \notin P$.

Hence Y is a T_1-GTMS space.

Note 8. *The converse of the above proposition is not true.*

It can be seen from the following example.

Example 5.3. *Let $Y = \{1,2,3\}$.*

We define generalized topology g_Y and minimal structure m_Y on Y as follows:

$g_Y = \{ \emptyset , \{1\} , \{3\} , \{1,3\} , \{1,2\} , \{2,3\} , Y \}$ *and*

$m_Y = \{ \emptyset , \{1\} , \{1,2\} , \{1,3\} , \{2,3\} , Y \}$.

Then Y is a T_1-GTMS space.

But not a T_2-GTMS pace.

Theorem 5.3. *Let (Y,g_Y,m_Y) be a GTMS space. Then the following conditions are equivalent :*

(1) Y is a T_2-GTMS space.

(2) $x \in Y$, then for any $y \neq x$, there exist a g_Y-open set O containing x such that $y \notin m_Y$-Cl(O).

(3) For all $x \in Y$, $\{x\} = \cap \{m_Y\text{-}Cl(O) : O \in g_Y$ and $x \in O\}$.

Proof

(1) implies (2)

Let us assume that Y *is a* T_2*-GTMS space and* $x \in Y$.

Let $y \in Y$ *be such that* $y \neq x$.

Then there exist a g_Y*-Open set* O *and a* m_Y*-Open set* P *such that*

$x \in O$, $y \in P$ *and* $O \cap P = \emptyset$.

Hence $y \notin m_Y$*-Cl(O).*

(2) implies (3)

Let $x \in Y$.

To prove that

$\{x\} = \cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\}$.

It follows that

$\{x\} \subseteq \cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\}$.

Let $y \in Y$ *be such that* $y \neq x$.

By our assumption, there exist a g_Y*-Open set* O_1 *containing* x *such that*

$y \notin m_Y$*-Cl(*O_1*).*

Then $y \notin \cap\{m_Y - Cl(O) : O \in g_Y andx \in O\}$.

Thus $\cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\} \subseteq \{x\}$.

Hence $\{x\} = \cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\}$.

(3) implies (1)

Let us assume that $\{x\} = \cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\}$ *for all* x.

Let $x, y \in Y$ *be such that* $x \neq y$.

Since $y \notin \{x\} = \cap\{m_Y - Cl(O) : O \in g_Y \text{ and } x \in O\}$.

Then there exists $O_1 \in g_Y$ *such that* $x \in O_1$ *and* $y \notin g_Y - Cl(O_1)$.

Since $y \notin O_1 - Cl(O_1)$ *, there exist* $P_1 \in m_Y$ *such that*

$y \in O_1$ *and* $O_1 \cap P_1 = \emptyset$.

Hence Y *is a* T_2*-GTMS space.*

Theorem 5.4. *Let (Y,g_Y,m_Y) be a GTMS space.*

Then the following conditions are equivalent :

(1) Y is a T_2-GTMS space.

(2) $x \in Y$, then for any $y \neq x$, there exist a m_Y-open set P containing x such that $y \notin g_Y$-$Cl(P)$.

(3) For all $x \in Y$, $\{x\} = \cap \{g_Y\text{-}Cl(O) : O \in m_Y$ and $x \in P\}$.

Proof

(1) implies (2)

Let us assume that Y is a T_2-GTMS space and $x \in Y$.

Let $y \in Y$ be such that $y \neq x$.

Then there exist a m_Y-open set P and a g_Y-Open set O such that

$x \in P$, $y \in O$ *and* $O \cap P = \emptyset$.

Hence $y \notin g_Y$-$Cl(P)$.

(2) implies (3)

Let $x \in Y$.

To prove that

$\{x\} = \cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\}$.

It follows that

$\{x\} \subseteq \cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\}$.

Let $y \in Y$ be such that $y \neq x$.

By our assumption, there exist a m_Y-Open set P_1 containing x such that

$y \notin g_Y$-$Cl(P_1)$.

Then $y \notin \cap\{g_Y - Cl(P) : P \in m_Y andx \in P\}$.

Thus $\cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\} \subseteq \{x\}$.

Hence $\{x\} = \cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\}$.

(3) implies (1)

Let us assume that $\{x\} = \cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\}$ *for all x.*

Let $x, y \in Y$ be such that $x \neq y$.

Since $y \notin \{x\} = \cap\{g_Y - Cl(P) : P \in m_Y \text{ and } x \in P\}$.

Then there exists $P_1 \in m_Y$ such that $x \in P_1$ and $y \notin m_Y - Cl(P_1)$.

Since $y \notin P_1 - Cl(P_1)$, there exist $O_1 \in g_Y$ such that

$y \in P_1$ and $P_1 \cap O_1 = \emptyset$.

Hence Y is a T_2-GTMS space.

Theorem 5.5. *A subspace of a T_2-GTMS space is T_2-GTMS .*

Proof

Let (Y,g_Y,m_Y) be a GTMS space.

Let Z be a subspace of Y.

Let b and c be two points of the subspace of Y.

If the neighbourhood O is different in Y of b then $O \cap Y$ is different neighbourhood of b in Z.

And if the neighbourhood P is different in Y of c then $P \cap Y$ is different neighbourhood of c in Z.

Let us consider the family of T_2-GTMS spaces be $\{Y_\alpha\}$.

Since $b \neq c$, there exist some index γ such that $Y_\gamma \neq z_\gamma$.

Let $b = (b_\alpha)$ and $c = (c_\alpha)$ be different points of the product space $\prod Y_\alpha$.

Take different open sets O and P in Y_γ containing Y_γ and Z_γ respectively.

Then $\prod^{-1}$(O) and $\prod^{-1}$(P) are different open sets in $\prod Y_\alpha$ contains b and c respectively.

Hence a subspace of a T_2-GTMS space is T_2-GTMS .

Definition 5.3. *Let (Y,g_Y,m_Y) be a GTMS space. A subset J of Y is said to be gmG-closed if g_Y-Cl(m_Y-Cl(J)) $\subseteq$ O whenever J $\subseteq$ O and O is open.*

Definition 5.4. *Let (Y,g_Y,m_Y) be a GTMS space. A subset J of Y is said to be mgG-closed if m_Y-Cl(g_Y-Cl(J)) $\subseteq$ O whenever J $\subseteq$ O and O is open.*

Definition 5.5. *A subset J of Y is said to be G-closed if J is mgG-closed and gmG-closed.*

Definition 5.6. *A GTMS sapce (Y,g_Y,m_Y) is called a $T_{1/2}$-GTMS space if every gmG-closed set is closed.*

Theorem 5.6. *Let (Y,g_Y,m_Y) be a GTMS space. Then the following are equivalent. (1) Y is a $T_{1/2}$-GTMS space.*
(2) Every singleton of Y is either open or closed.

Proof

(1) implies (2)

Let Y be a $T_{1/2}$-GTMS space and $y \in Y$.

If Y is not closed then Y-$\{y\}$ is not open and then Y-$\{y\}$ is trivially gmG-closed.

Thus Y-$\{y\}$ is closed and hence $\{y\}$ is open.

(2) implies (1)

Let J be a gmG-closed and $x \in g_Y$-Cl(m_Y-Cl(J)).

We have the following two cases:

Case I :

Suppose $\{y\}$ is closed.

Then g_Y-Cl(m_Y-Cl(J))-J does not contain any nonempty closed set.

This shows that $y \in J$.

Case II :

Suppose $\{y\}$ is open.

If $y \notin J$, then $J \subseteq$ Y-$\{y\}$.

Since Y-$\{y\}$ is closed, then g_Y-Cl(m_Y-Cl(J)) $\subseteq$ g_Y-Cl(m_Y-Cl(Y-$\{y\}$)) = Y-$\{y\}$.

Thus $y \notin g_Y$-Cl(m_Y-Cl(J)).

In either case, g_Y-Cl(m_Y-Cl(J)) = J, that is J is closed.

Thus Y is a $T_{1/2}$-GTMS space.

Theorem 5.7. *Let (Y,g_Y,m_Y) be a GTMS space. Then the following are equivalent. (1) Y is a $T_{1/2}$-GTMS space.*

(2) Every mgG-closed set is closed.

Corollary 5.1. *Let (Y,g_Y,m_Y) be a GTMS space. Then the following hold.*

(1) If Y is a $T_{1/2}$-GTMS space, then subset J of Y is gmG-closed if and only if J is mgG-closed.

(2) Y is a $T_{1/2}$-GTMS space if and only if every G-closed set is closed.

Theorem 5.8. *Let (Y,g_Y,m_Y) be a $T_{1/2}$-GTMS space. Then the following hold.*

(1) Every gG-closed set is closed.

(2) Every singleton of Y is either open or g-closed.

Proof

(1) Suppose that J is gG-closed.

Then J is mgG-closed.

Since Y is $T_{1/2}$-GTMS space, then J is closed.

(2) Let $y \in$ Y.

If y is not g-closed, then Y-$\{y\}$ is not g-open and then Y-$\{y\}$ is trivially gG-closed.

By (1), Y-$\{y\}$ is closed and hence $\{y\}$ is open.

Note 9. *The converse of the above theorem is need not be true, in general, as a simple we give the following example.*

Example 5.4. *Let Y = $\{1,2,3\}$ with generalized topology g = $\{\emptyset$, Y, $\{1\}$,$\{1,2\}$, $\{1,3\}$ $\}$ and minimal structure m = $\{\emptyset$, Y, $\{1\}$,$\{2\}$ $\}$.*

It is easy to check that every gG-closed set is closed and every singleton of Y is either open or g-closed.

But Y is not $T_{1/2}$-GTMS.

Theorem 5.9. *A GTMS space (Y,g_Y,m_Y) is a $T_{1/2}$-GTMS space if and only if for any singleton $\{y\}$ in Y there exists either s-closed subset J such that $\{y\}$ = g_Y-Cl(J) or s-open subset K such that $\{y\}$ = g_Y-Int(K).*

Proof

Let us assume that Y is a $T_{1/2}$-GTMS space .

Then $\{y\}$ is closed or open.

If $\{y\}$ is closed, then there exists a s-closed subset J such that $\{y\}$ = g_Y-Cl(J).

On the other hand, let $\{y\}$ be open.

That is Y-$\{y\}$ is closed.

Then there exists a s-closed subset L such that Y-$\{y\}$ = g_Y-Int(L).

Thus $\{y\}$ = g_Y-Int(Y-L).

Put K = Y-L, we have a s-open subset K such that $\{y\}$ = g_Y-Int(K).

Conversely,

Case I :

There exists a s-closed subset K such that $\{y\}$ = g_Y-Cl(J).

Then $\{y\}$ is closed.

Case II :

There exists a s-open subset K such that $\{y\}$ = g_Y-Int(K).

Thus Y-$\{y\}$ = Y-(g_Y-Int(K)) = g_Y-Cl(Y-K).

Then Y-$\{y\}$ is closed, since Y-K is s-closed.

Thus $\{y\}$ is open.

The two cases show that $\{y\}$ is either closed or open.

This shows that Y is $T_{1/2}$-GTMS space.

Theorem 5.10. *A GTMS space (Y,g_Y,m_Y) is a $T_{1/2}$-GTMS space if and only if for any singleton $\{y\}$ in Y there exists either c-closed subset J such that $\{y\}$ = g_Y-Cl(m_Y-Cl(J)) or c-open subset K such that $\{y\}$ = g_Y-Int(m_Y-Int(K)).*

Theorem 5.11. *Let (Y,g_Y,m_Y) be a $T_{1/2}$-GTMS space and $y \in Y$.*
If $\{y\}$ is not g-closed, then the following hold.
(1) There exists a s-open subset J such that $\{y\}$ = g_Y-Int(J).
(2) There exists a c-open subset J such that $\{y\}$ = g_Y-Int(m_Y-Int(J)).

Theorem 5.12. *Let (Y,g_Y,m_Y) be a GTMS space. If Y is a T_1-GTMS space then Y is a $T_{1/2}$-GTMS*

Note 10. *The converse of the above theorem is need not be true, in general, as a simple we give the following example.*

Example 5.5. *Let $Y = \{1,2,3\}$ with generalized topology $g = \{\emptyset,\ Y,\ \{1\}\ ,\{3\},\ \{1,3\}\ \}$ and minimal structure $m = \{\emptyset,\ Y,\ \{1\}\ ,\{3\},\ \{1,2\},\ \{1,3\}\}$. Then Y is a $T_{1/2}$-GTMS but not T_1-GTMS space.*

Chapter 6

CONCLUSION

We introduced the concepts of m_Y-open sets, m_Y-closed sets,m_Y-closure and $m_Y - interior$ operators with their properties and also we introduced the concepts of T_1-GTMS space T_2-GTMS space and $T_{1/2}$-GTMS space. Further this theory can also be developed in soft topological space and Fuzzy topological space.

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