Research Proposal

By

G. Chitra

Part Time Ph.D Programme at Kalasalingam Academy for Research and Education-KARE, Deemed to be University, Krishankoil, TN, India Pincode-626126

Title of my Ph.D Thesis Work: *Concept of Domination number in Graphs and Its Applications*

Area: Graph Theory **Sub Area**: Domination Theory in Graphs

Investigation of the concept of domination in graphs began around 1960.

History

Even though it began around 1960, still some references point to related problems a century earlier. In 1862, de Jaenisch [1] endeavored to determine the least number of queens necessary to cover an $n \times n$ chess board. W. W. Rouse Ball in 1892 [2] recorded three problems that chess players considered during this time. a. Covering: Find the least number of chess pieces of a given type that are required to cover every square of an $n \times n$ chess board. b. Independent Covering: Find the least number of mutually non-attacking chess pieces of a given type that are required to dominate every square of an $n \times n$ board. c. Independence: Find the greatest number of chess pieces of a given type that can be placed on an $n \times n$ chess board such that no two pieces attack each other. Investigation of domination in graphs was grew further in the late 1950's and 1960's, starting with Claude Berge [3] in 1958. He wrote a book in which he coined the term "coefficient of external stability," now acknowledged as the domination number of a graph. Oystein Ore [4] framed the terms "dominating set" and "domination number". Yaglom and Yaglom brothers further developed it in 1964 [5]. Cockayne and Hedetniemi [6] published a survey paper, in which the notation (G) was first used for the domination number of a graph G.

I propose to work on Vizing's conjecture that states: The domination number of the Cartesian product of two graphs is at least the product of their domination numbers. It still remains as one of the long standing open problems. A number of partial results can be seen in literature, but it is yet to be proved in general. In 2014, me and Yegnanarayanan computed for paths and cycles certain graph domination invariants like locating domination number, differentiating domination number, global alliance number etc., We also did some comparison analysis of certain parameters defined by combining the

domination measures and the second smallest eigen value of the Laplacian matrix of all connected graphs of order 4. While discussing applications we have pointed out the crucial role played by graphs through its hard core structural properties in wireless sensor networks (WSN) [7]. W propose to improve the lower bound on distance domination number of G with reference to the diameter and minimum degree and the upper bound as well with reference to the order and minimum k distance neighborhood.

Given a graph G one can partition the vertex set of G, V(G) into k disjoint sets represented as $\pi = \{V_1, V_2,...,V_k\}$. Call a set X of vertices dominates another set of vertices, Y, if for every vertex $y \in Y$ there exists some vertex $x \in X$ adjacent to y. The upper domatic number of a graph G denoted D(G) and defined as the maximum integer k such that G can be partitioned into k sets where for every pair of sets V_i, $V_j \in \pi$ either V_i dominates V_j or V_j dominates V_i or both. We propose to attempt the following open problems.

a. Is there a graph G with $|V(G)| < 10$ where $D(G) < \alpha(G)$?
b. What is the smallest $\delta(G)$ such that $D(G) < \delta(G)$?
c. Is there a high level algorithm for finding an upper domatic partition of a graph?
d. How much larger can D(H) be than D(G) where H is a subgraph of G?
e. Do there exist graphs with no D-partition that contains a sink set?
f. What is the time complexity of deciding if a given graph has $D(G) \geq 4$? $D(G) \geq 5$?
g. What classes of graphs can be described as "upper domatically full," if any?

Then, Wireless sensor networks (WSNs) includes spatially distributed autonomous sensor nodes to monitor environmental conditions and hence they have widespread applications in human communities and existence world. We intend to probe topology control and virtual backbone based on dominating sets for wireless sensor networks and deal with the performance evaluation of connected dominating set and other variations and develop construction algorithms for the same. I wish to proceed further and probe brain networks and social networks as well.

Bibliography

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