# Introduction

Continued Fractions are important in many branches of mathematics. They arise naturally in long division and in the theory of approximation to real numbers by rationals. These objects that are related to number theory help us find good approximations for real life constants.

## Euclid's GCD algorithm

Given two positive integers, this algorithm computes the greatest common divisor (gcd) of the two numbers.

**Algorithm:** Let the two positive integers be denoted by $a$ and $b$.

1. If $a < b$, swap $a$ and $b$.
2. Divide $a$ by $b$ and find remainder $r$. If $r = 0$, then the gcd is $b$.
3. If $r$ *not equal to zero*, then set $a = b$, $b = r$ and go back to step 1.

This algorithm terminates and we end up finding the gcd of the two numbers we started with.

**EXAMPLE:**

Take a=43 and b=19

$$\begin{aligned} 43 &= \mathbf{2} \times 19 + 5 \\ 19 &= \mathbf{3} \times 5 + 4 \\ 5 &= \mathbf{1} \times 4 + 1 \\ 4 &= \mathbf{4} \times 1 + 0 \end{aligned}$$

Hence, by Euclid's algorithm, the gcd of 43 and 19 is 1.

Observe that the quotient at each step of the algorithm has been highlighted. Using these numbers we can present the fraction in the following manner:

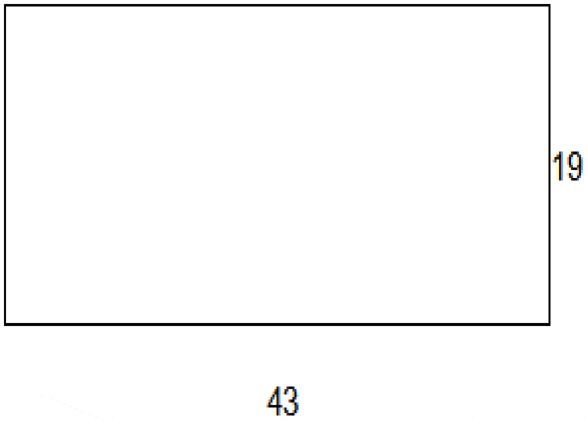
$$\frac{43}{19} = 2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4}}}$$

In general it is true that given two positive integers, we can write the fraction in the above format by using the successive quotients obtained from Euclid's algorithm.

## Pictorial Description

Lets look at the same example in a pictorial manner.

Consider a rectangle whose length is 43 units and whose width is 19 units.



$$\frac{43}{19} = 2 + \cfrac{1}{3 + \cfrac{1}{1 + \cfrac{1}{4}}}$$

## Simple Continued Fraction

**Definition:** A Simple Continued Fraction is an expression of the form

$$= a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \dots}}}$$

where $a_i$ are non-negative integers, for $i > 0$ and $a_0$ can be any integer.

The above expression is cumbrous to write and is usually written in one of these two forms:

$$= a_0 + \frac{1}{a_1 +} \frac{1}{a_2 +} \frac{1}{a_3 +} \cdots$$

Or

using the list notation

$$[a_0, a_1, a_2, \ldots]$$