Introduction

Can the special structure possessed by social networks be exploited algorithmically? Answering this question requires a formal definition of social network structure. The most often mentioned (and arguably most validated) statistical properties of social networks include heavy-tailed degree distributions [1,2,7], a high density of triangles [23,27,29] and other dense subgraphs or communities [10, 11, 17, 20, 21], and low diameter and the small world property [14–16, 19]. Much of the recent mathematical work on social networks has focused on the important goal of developing generative models that produce random networks with many of the above statistical properties. However, the paper [12] of R. Gupta, T. Roughgarden and C. Seshadhri pursues a different approach. In lieu of adopting a particular generative model for social networks, they ask, Is there a combinatorial assumption weak enough to hold in every reasonable model of social networks, yet strong enough to permit useful structural and algorithmic results? Specifically, we seek structural results that apply to every reasonable model of social networks, including those yet to be devised.

Some Results

In the paper [12], they give results about triangle-dense graphs and everywhere triangle-dense graphs. Let a wedge be a 2-path in an undirected graph. The following are the definitions.

Definition (triangle-dense graph) The triangle density of an undirected graph G = (V, E) is τ (G) := 3t(G)/w(G), where t(G) is the number of triangles in G and w(G) is the number of wedges in G (conventionally, τ (G) = 0 if w(G) = 0). The class of -triangle-dense graphs consists of the graphs G with τ (G) ≥ . Since every triangle of a graph contains three wedges, and no two triangles share a wedge, the triangle density of a graph is between 0 and 1. As an example, the triangle density of a graph is 1 if and only if it is the union of cliques. The triangle density of an Erd¨os-Renyi graph, drawn from G(n, p), is concentrated around p. Thus, only dense Erd¨os-Renyi graphs have constant triangle density (as $n \to \infty$). Social networks are generally sparse and yet have remarkably high triangle density; the Facebook graph, for instance, 1 has triangle density 0.16 [27]. The class of -triangle-dense graphs becomes quite diverse as soon as is bounded below 1. For example, the complete tripartite graph is triangle dense with ≈ 1 2 .