OPTIMIZATION METHODS FOR LARGE-SCALE NONLINEAR NONCONVEX CONSTRAINED OPTIMIZATION

Assessment Title: Research Proposal

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Introduction

Large-scale nonlinear optimization is concerned with the numerical solution of continuous problems.An optimization problem is nonlinear if the objective function f(x) or any of the inequality constraints $c_i(x) \leq 0$, i = 1, 2, …, m, or equality constraints $d_j(x) = 0$, j = 1, 2, …, n, are nonlinear functions of the vector of variables x.

Nonlinear programming algorithms typically proceed by making a sequence of guesses of the variable vector x (known as iterates and distinguished by superscripts x^1, x^2, x^3, …) with the goal of eventually identifying an optimal value of x. Often, it is not practical to identify the globally optimal value of x

In mathematical optimization, constrained optimization (in some contexts called constraint optimization) is the process of optimizing an objective function with respect to some variables in the presence of constraints on those variables.

Constrained optimization problems are problems for which a function is to be minimized or maximized subject to constraints . Here is called the objective function and is a Boolean-valued formula.

Constraints can be either hard constraints, which set conditions for the variables that are required to be satisfied, or soft constraints, which have some variable values that are penalized in the objective function if, and based on the extent that, the conditions on the variables are not satisfied.

A non-convex optimization problem is any problem where the objective or any of the constraints are non-convex, as pictured below. Such a problem may have multiple feasible regions and multiple locally optimal points within each region

A non-convex function need not be a concave function. For example, the function $f(x)=x(x-1)(x+1)$ defined on $[-1,1]$.

An OptimizationProblem object describes an optimization problem, including variables for the optimization, constraints, the objective function, and whether the objective is to be maximized or minimized. Solve a complete problem using solve . Tip. For the full workflow, see Problem-Based Optimization Workflow

Non-convex optimization is a type of mathematical optimization problem in which the objective function to be optimized is not convex. Unlike convex optimization problems, non-convex problems can have multiple local optima, which can make it difficult to find the global optimum. Non-convex optimization has many applications in various fields, including finance (portfolio optimization, risk management, and option pricing) [1,2,3], computer vision (image segmentation, object recognition) [4,5], signal processing (compressed sensing, channel estimation, and equalization) [6,7,8], engineering (control systems, optimization of structures) [9,10], machine learning [11,12,13,14], and damage characterization [15] based on deep neural networks and the YUKI algorithm [16].

To solve non-convex optimization problems, two broad classes of techniques have been developed: deterministic and stochastic methods [17]. Deterministic methods include gradient-based methods, which rely on computing gradients of the objective function, and which can be sensitive to the choice of initialization and can converge to local optima. On the other hand, stochastic methods use randomness to explore the search space and can be less sensitive to initialization and more likely to find the global optimum.

Literature Review

Constrained optimization, also known as constraint optimization, is the process of optimizing an objective function with respect to a set of decision variables while imposing constraints on those variables.

There are several applications for nonlinear programming. Some of the most common are engineering design, control, data fitting, and economic planning. These applications usually share some attributes regarding problem structure that make convex optimization algorithms very effective.

Nonlinear optimization problems have a wide range of applications in many different fields, including engineering, finance, and data analysis. For example, nonlinear optimization problems may be used to: Design efficient and cost-effective systems, such as electrical power grids or transportation networks

The advantage of this method is to transform the constrained optimization problem into an unconstrained one. That is, all the constraints are incorporated into the new objective function. However, this introduces more free parameters whose values need to be defined so as to solve the problem appropriately.

Nonlinear programming algorithms are used to solve optimization problems in which either the objective function or the constraints are modelled using nonlinear functions of the decision variables, and all are given as explicit mathematical functions

Constrained optimization methods are informative in providing insights to decision makers about optimal target solutions and the magnitude of the loss of benefit or increased costs associated with the ultimate clinical decision or policy choice.

Constrained optimization is a powerful technique that can be used to optimize various processes and operations subject to constraints. By finding the optimal solution to a problem while satisfying constraints, it can help decision-makers make informed decisions, allocate resources efficiently, and reduce costs.

Constrained optimization methods provide a structured approach to optimize the decision problem and to present the best alternatives given an optimization criterion, such as constrained budget or availability of resources.

There are two ways to solve nonlinear optimization problems in MATLAB: using a problem-based approach or a solver-based approach. This example uses a problem-based approach, which uses optimization variables to define the objective and constraints. See the documentation for the solver-based approach.

There are two major complexity models for nonlinear optimization. One that seeks to approximate the objective function. The second, which we present here, approximates the optimal solution in the solution space.

These methods include the substitution method and the elimination method. Other algebraic methods that can be executed include the quadratic formula and factorization.

The theory of nonlinear systems has applications to problems of population growth, economics, chemical reactions, celestial mechanics, physiology of nerves, onset of turbulence, regulation of heartbeats, electronic circuits, cryptography, secure communications and many others.

The triple constraint triangle of project management is the visualization of a triangle, with sides formed by time, scope, and cost.

Research Questions

- What is the best way for solving non-linear optimization problems when you are looking for global optimum?
- What is the impact factor of nonlinear analysis real world applications?
- What's more difficult to optimize non-convex or convex?
- What is the importance of constrained optimization in business management?

Research Methodology

Over the years, several techniques have been developed for the solution of non linear programming problems and some of the prominent techniques are: method of feasible directions, sequential unconstrained minimization technique, sequential linear programming and dynamic programming.

The first step is to transform the nonlinear objective function into a Lagrangian function. This is accomplished by transforming the constraint equation as follows. Next, this expression is multiplied by , the Lagrangian multiplier, and subtracted from the objective function to form the Lagrangian function L.

To set up a nonlinear optimization problem for solution, first decide between a problem-based approach and solver-based approach. See First Choose Problem-Based or Solver-Based Approach. For problem-based nonlinear examples and theory, see Problem-Based Nonlinear Optimization.

The Newton method is one of the best methods to determine the root solution of nonlinear equations (Sánchez 2009). In its development the Newton method is also used to find the optimum point of an optimization problems (Silalahi 2014).

For NCO, many CO techniques can be used such as stochastic gradient descent (SGD), mini-batching, stochastic variance-reduced gradient (SVRG), and momentum. There are also specialized methods for solving non-convex problems known in operations research such as alternating minimization methods, branch-and-bound methods.

local optimization of the non-convex function All convex functions rates apply. ○ rescales gradients by the absolute value of the inverse Hessian and the Hessian's Lanczos vectors. ○ matrix completion ○ Image reconstruction ○ recommendation systems.

The commonly used mathematical technique of constrained optimizations involves the use of Lagrange multiplier and Lagrange function to solve these problems followed by checking the second order conditions using the Bordered Hessian.

As a result, the method of Lagrange multipliers is widely used to solve challenging constrained optimization problems. Further, the method of Lagrange multipliers is generalized by the Karush–Kuhn–Tucker conditions, which can also take into account inequality constraints of the form for a given constant .

Time table

YEAR ONE	Literature Review Data Collection
YEAR TWO	Data Analysis- Statistical, Computational and other Thesis Writing
YEAR THREE	Write Up Drafting a thesis Thesis Completion

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