"A Unique Visualisation of L-fuzzy Sets in RL-fuzzy Bitopological Spaces"

1. Introduction

In 1963, Levine [1] introduced the notion of semiopen set and its corresponding associated function in the realm of general topology. Afterwards, Azad [2] extended this notion and its related functions to the setting of L-topology. Thakur and Malviya [3] introduced and studied the concepts of (i, j)-semiopen and (i, j)-semiclosed L-fuzzy sets, pairwise fuzzy semicontinuous, and pairwise fuzzy semiopen functions in L-bitopology in the case of $L = [0, 1]$. In [4], Shi introduced the notion of L-fuzzy semiopen and preopen gradations in L-fuzzy topological spaces. Furthermore, he introduced the notions of L-fuzzy semicontinuous functions, L-fuzzy pre continuous functions, L-fuzzy irresolute functions, and L-fuzzy pre-irresolute functions, and discussed some of their elementary properties. Shi's operators have been found very useful in defining other gradations and also in studying many topological characteristics. In 2011, Ghareeb [5] used L-fuzzy preopen operator to introduce the degree of pre-separatedness and the degree of pre connectedness in L-fuzzy topological spaces. Many characterisations of the degree of pre connectedness are discussed in L-fuzzy topological spaces. Later, Ghareeb [6] introduced the concept of L-fuzzy semi-preopen operator in L-fuzzy topological spaces and studied some of its properties. The concepts of L-fuzzy SP-compactness and L-fuzzy SP-connectedness in L-fuzzy pre topological spaces are introduced and studied [7]. Further, a new operator in L-fuzzy topology introduced in [8] to measure the **F**-openness of an L-fuzzy set in L-fuzzy topological spaces. Moreover, the new operator is used to introduce a new form of **F**-compactness. Recently, we used the new operators to generalize several kinds of functions between L-fuzzy topological spaces [9–12].

Recently, Li and Li [13] defined and studied the concept of RL-topology as an extension of L-topology. Moreover, RL-compactness by means of an inequality and RL-continuous mapping are introduced and discussed in detail. In [14], they presented RL-fuzzy topology on an L-fuzzy set as a generalisation of RL-topology and L-fuzzy topology. Some relevant properties of RL-fuzzy compactness in RL-fuzzy topological spaces are further investigated. Later on, Zhang et al. [15] defined the degree of Lindelöf property and the degree of countable RL-fuzzy compactness of an L-fuzzy set, where L is a complete DeMorgan algebra. Since L-fuzzy topology in the sense of Kubiak and Šostak is a special case of RL- fuzzy topology, the degree of RL-fuzzy compactness and the degree of Lindelöf property are extensions of the corresponding degrees in L-fuzzy topology.

2. Research Objectives

The primary objectives of this research are:

1. Based on the idea of pseudo-complement of L-fuzzy sets, the (i, j)-RL-semiopen gradation in RL-fuzzy bitopological spaces is introduced in this study.
2. Pairwise RL-fuzzy semicontinuous, pairwise RL-fuzzy irresolute functions, and pairwise RL-fuzzy semi-compactness are also defined and described.
3. Compared to the analogous notions in L-bitopology, RL-bitopology, RL-fuzzy topology, L-fuzzy topology, and L-fuzzy bitopology, our findings are more general

3. Literature Review

Introduced by Zadeh in 1965, fuzzy set theory offers a mathematical framework for dealing with imprecision and uncertainty. It has been widely used in many applied and mathematical fields. To broaden the membership functions using a lattice structure, L-fuzzy sets—an extension of fuzzy sets were presented (Goguen, 1967). These sets are especially helpful in topology, control systems, and decision-making because they provide more flexibility in modelling graded uncertainty.

Two distinct topologies on a given set are studied in bitopological spaces, which were first proposed by Kelly in 1963. There are several computational and analytical applications for this dual-topology structure. Fuzzy bitopological spaces, where fuzzy topologies are assigned to two distinct structures on the same set, are the result of the fusion of fuzzy logic and bitopology (Lowen, 1976). This idea is further developed by RL-fuzzy bitopological spaces, which incorporate L-fuzzy sets into bitopological structures. Applications for this approach can be found in fuzzy decision-making, pattern recognition, and computational intelligence.

It is difficult but crucial to see L-fuzzy sets in order to comprehend their structural characteristics. New visualization techniques have emerged as a result of recent developments in computational techniques like graphical modeling and machine learning. Scholars have investigated many methods for graphically representing fuzzy topological structures, including heat-maps, contour plots, and three-dimensional mappings (Mizoguchi & Shinohara, 1990). These techniques aid in improved L-fuzzy topology interpretation and analysis.

The concept of RL-fuzzy bitopological spaces has been applied in various domains, including decision theory, artificial intelligence, and mathematical modeling. These spaces allow a more refined classification of elements based on two different topological perspectives. Recent studies have demonstrated their effectiveness in uncertainty modeling, particularly in economic and medical decision-making scenarios (Mahapatra & Samanta, 2015).

Even while the study of L-fuzzy sets and their applications in bitopological spaces has advanced significantly, there are still a number of unanswered questions. More sophisticated computational approaches can be investigated for improved representation as the visualization techniques for these structures are still in their infancy. Furthermore, more theoretical work is required to integrate L-fuzzy sets with contemporary computational frameworks like deep learning and quantum computing.

4. Conclusion

The study of L-fuzzy sets in RL-fuzzy bitopological spaces is a rapidly evolving field with significant theoretical and practical implications. The unique visualization of these sets provides better insights into their structure and behavior. Future research should focus on refining visualization techniques and exploring new applications in artificial intelligence and computational mathematics.

5. References

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