



KALASALINGAM
ACADEMY OF RESEARCH AND EDUCATION
(DEEMED TO BE UNIVERSITY)
Under sec. 3 of UGC Act 1956. Accredited by NAAC with "A++" Grade



DISSERTATION PROJECT

SYNOPSIS

TOPIC: A Unified Framework for Anti-Fuzzy Soft Sets and Their Applications in Multi-Criteria Decision-Making



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"A Unified Framework for Anti-Fuzzy Soft Sets and Their Applications in Multi-Criteria Decision-Making"

1. Introduction

In real-world decision-making scenarios—such as medical diagnosis, risk management, supplier selection, and engineering design—information is often uncertain, vague, incomplete, and even contradictory. Classical set theory, which uses crisp membership (an element either belongs or does not belong to a set), fails to model such uncertainty. To overcome this, Lotfi A. Zadeh (1965) introduced fuzzy set theory, where elements have partial membership values between 0 and 1. Over time, several extensions such as intuitionistic fuzzy sets, bipolar fuzzy sets, and neutrosophic sets were developed to handle hesitation, dual polarity, and indeterminacy. However, all these models mainly capture positive degrees of belonging and cannot explicitly express the extent to which elements oppose or contradict membership. This limitation led to the emergence of Anti-Fuzzy Set Theory, which introduces an anti-membership function representing the degree of opposition to inclusion in a set. The concept has been explored primarily in algebraic contexts—like anti-fuzzy subgroups, ideals, and semirings—but lacks a unified mathematical foundation and practical applications. Parallely, Soft Set Theory (Molodtsov, 1999) offers a parameterized approach to uncertainty, enabling flexible representation of decision parameters but without capturing contradictory evidence. Therefore, integrating the dual nature of anti-fuzziness with the parameterization of soft sets can lead to a powerful framework capable of modeling both supporting and opposing evidence under uncertainty. Such a unified Anti-Fuzzy Soft Set (AFSS) model can significantly enhance decision-making systems by capturing multidimensional, conflicting criteria more realistically. This research thus aims to develop a comprehensive theoretical structure for anti-fuzzy soft sets, define their fundamental properties and operations, and design decision-making algorithms based on this framework to handle complex multi-criteria problems where both positive and negative evidence coexist.

2. Research Objectives

The primary objective of this research is to develop a unified theoretical and computational framework for Anti-Fuzzy Soft Sets (AFSS) and to demonstrate their applicability in modeling uncertainty and contradiction within multi-criteria decision-making environments. In alignment with this aim, the specific research objectives are as follows:

- 1. To formulate a generalized definition of Anti-Fuzzy Soft Sets (AFSS)** by integrating the fundamental principles of fuzzy, anti-fuzzy, and soft set theories into a single coherent mathematical structure.
- 2. To establish and analyze the core operations and properties** of AFSS—such as union, intersection, complement, equality, inclusion, and product—and to prove related algebraic theorems ensuring internal consistency.
- 3. To investigate the relationships and mappings** between AFSS and existing uncertainty models, including fuzzy soft sets, intuitionistic fuzzy soft sets, and neutrosophic soft sets, thereby identifying their similarities, extensions, and special cases.
- 4. To construct a decision-making framework** based on the AFSS model that effectively handles positive and opposing evidence in multi-criteria decision problems.
- 5. To design and implement computational algorithms** (such as anti-fuzzy soft aggregation, ranking, and score functions) for solving decision problems involving conflicting or contradictory criteria.

6. **To validate the proposed AFSS-based decision model** through real-world or simulated case studies (e.g., medical diagnosis, risk assessment, supplier selection) and perform comparative analysis with classical fuzzy and neutrosophic approaches.
7. **To explore possible extensions and generalizations** of the AFSS framework toward advanced hybrid models, such as anti-neutrosophic soft sets, anti-fuzzy graphs, and anti-fuzzy decision systems.

3. Literature Review

Fuzzy set theory, which introduced graded membership to model vagueness and imprecision, is the classical foundation for all degree-based uncertainty models; Lotfi A. Zadeh's seminal 1965 paper remains the canonical reference for fuzzy sets and their basic operations and properties. Soft set theory, proposed by Molodtsov (1999), introduced a complementary direction by providing a parameterized framework for handling uncertainties that are difficult to model by probability or fuzzy membership alone; since then many hybrid models (fuzzy-soft, intuitionistic fuzzy-soft, neutrosophic-soft, etc.) have been proposed to combine parameterization with graded uncertainty.

The term anti-fuzzy appears across a growing but still specialized literature, mostly within algebraic-structure research. Early/representative works formalized anti-fuzzy variants of algebraic objects (e.g., anti-fuzzy subgroups, anti-fuzzy ideals) and developed theorems about their closure, level sets, and homomorphic images; Gayen and collaborators provide notable generalizations and structural results on anti-fuzzy subgroups. Researchers have extended anti-fuzzy ideas to more complex algebraic contexts: for example, anti-fuzzy soft subhemirings were proposed in investigations of hemirings and soft algebraic structures (Anitha & Venkatesan, 2017), and anti-fuzzy multi-ideals of near-rings were developed to link fuzzy multisets with near-ring algebraic theory (Hošková-Mayerová & Al Tahan, 2021). These works carefully define anti-membership notions in their respective algebraic contexts and prove families of structural theorems.

Other algebraic studies have explored anti-fuzzy ideals in non-associative structures (e.g., AG-groupoids) and interior/left/right anti-fuzzy ideals in ordered algebraic systems; collectively these papers demonstrate that anti-fuzzy constructs are mathematically robust across a range of algebraic settings but are still treated largely as theoretical generalizations rather than as components of broader, unified frameworks. Survey and review treatments of soft set theory and its many hybrids show that soft sets are frequently used for decision support, medical diagnosis, and economic modeling, but systematic incorporation of an explicit anti-membership or opposition component within soft set frameworks is absent in major surveys—highlighting a methodological gap between parameterized decision models and algebraic anti-fuzzy research. A few recent papers explore specialized anti-fuzzy notions beyond simple complements (for example, complex anti-fuzzy isomorphisms and refined anti-fuzzy subgroup concepts), indicating ongoing interest and diversification of definitions; however, the literature shows no widely adopted unified definition that integrates anti-fuzziness with parameterized soft sets and very few applied decision-making frameworks that exploit anti-membership to model conflicting evidence.

(1) lack of a standardized, general AFSS definition that subsumes existing anti-fuzzy algebraic variants; (2) limited study of AFSS operations, algebraic/topological properties, and inter-model mappings (e.g., relations to intuitionistic or neutrosophic soft sets); (3) almost no end-to-end MCDM frameworks or computational algorithms exploiting anti-fuzziness plus parameterization; and (4) scarce empirical comparisons between anti-fuzzy approaches and established fuzzy/neutrosophic decision models. These gaps motivate your proposed research to build a unified theory (definitions + theorems) and practical AFSS-based decision algorithms validated on real or simulated case studies. (

4. Conclusion

In summary, the proposed research seeks to bridge a significant theoretical and practical gap in the domain of uncertainty modeling by introducing a unified framework for Anti-Fuzzy Soft Sets (AFSS). While fuzzy and soft set theories have proven highly effective in addressing vagueness and parameterized uncertainty, they remain limited in their ability to represent opposition, contradiction, and negative evidence explicitly. Existing studies on anti-fuzzy structures, though mathematically rich, are largely confined to isolated algebraic contexts and lack integration with modern decision-making frameworks. This research, therefore, aims to establish a comprehensive theoretical foundation for AFSS, rigorously defining its operations, properties, and interrelations with other uncertainty models such as fuzzy, intuitionistic, and neutrosophic soft sets. Beyond theory, the study will develop computational algorithms and a decision-making model capable of processing both supporting and opposing information, thereby enhancing analytical accuracy and interpretability in real-world applications. The expected outcome is a robust mathematical and computational system that not only extends current uncertainty theories but also provides new pathways for intelligent decision support in domains such as engineering, healthcare, and risk assessment. Ultimately, this research will contribute to advancing the frontiers of soft computing, fuzzy logic, and decision sciences by formalizing and applying the underexplored concept of anti-fuzziness within a coherent soft set environment.