

Radical Quadratic and Holistic Prime Labeling of Graphs

1. Introduction

In this work, I introduce and study two new type of labeling:

- Radical Quadratic Labeling
- Holistic Prime Labeling

In this paper I define Radical Quadratic and Holistic Prime Labeling of Graph and establish the relation between Radical Quadratic and Holistic Prime Labeling.

2. Basic Concepts

2.1 Radical Quadratic Labeling

Let $G = (V, E)$ be a simple graph. A one - to - one function $f: V(G) \rightarrow \{1, 2, \dots, k\}$ is called a **Radical Quadratic Labeling** if for every edge $ab \in E(G)$, $f(ab) = \lfloor \sqrt{f(a)^2 + f(b)^2} / 2 \rfloor$ and all edge labels $f(ab)$ are distinct.

2.2. Holistic Prime Labeling

A function $g: V(G) \cup E(G) \rightarrow \{1, 2, \dots, p+q\}$ is said to be a **Holistic Prime Labeling** if for every edge $uv \in E(G)$, $\gcd(g(a), g(b), g(ab)) = 1$. That is the labels of the two vertices and their connecting edge are mutually coprime.

3. Methodology

Review of Literature: Study existing works on root square mean, and total prime labeling.

Definition and Formulation: Define the new labelings formally.

Example Construction: Start with small graphs P_n to test labeling.

Theoretical Proofs: Prove that certain classes of graphs admit these labeling.

Generalization: Extend results to larger and compound graphs (like corona and product graphs).

4. Expected Results

Development of new labeling definitions and theorems.

Proof that several graph families admit radical quadratic and holistic prime labeling.

Establishment of connections between root square mean and radical quadratic labeling.

Identification of graphs that do not admit such labeling.

Formulation of conjectures for further research.

5. Conclusion

The study will provide new theoretical results, examples, and applications that can serve as a foundation for further developments in graph labeling theory.